IDENTIFYING THE EFFECTS OF STIFFNESS CHANGES IN A 5-DOF SYSTEM

Alan Brockman[1], Joshua Clough[2], Kirun Mulji[3], Norman Hunter[4]

[1]Dept. of Mechanical Engineering, Purdue University. West Lafayette, IN 47907
[2]Dept. of Mechanical Engineering, Rose-Hulman Inst. of Technology. Terre Haute, IN 47803
[3]Dept. of Civil Engineering, Univ. of California Irvine. Irvine, CA 92697
[4]Measurement Technology Group, Los Alamos National Laboratory. Los Alamos, NM 87545

ABSTRACT

Using a system of five masses and four springs, both linear and non-linear changes in stiffness were detected by examining the frequency and time response of the system. The replacement of an individual spring with one of a different stiffness value created a linear change, while nonlinearities were introduced through the use of collisions between masses. From the time history of the input force and the accelerations of each mass, the frequency response functions, natural frequencies, mode shapes, power spectra, and probability density functions were calculated. These results were used, in conjunction with a numerical model, to detect changes in the system. In general, the natural frequencies and mode shapes were the best identifiers for linear changes, while the power spectra and probability density functions best identified nonlinear changes.

NOMENCLATURE

[C]	Viscous damping matrix
DOF	Degree-of-freedom
[F]	Input force vector
[K]	Stiffness matrix
FFT	Fast Fourier transform
FRF	Frequency response function
[M]	Mass matrix
PDF	Probability density function
TFE	Transfer function estimator
X	Position vector

1 INTRODUCTION

With the increased complexity of today's dynamic systems, non-destructive damage detection in multiple DOF systems is crucial. Damage detection based on vibration response could be used for evaluating buildings, bridges, aircraft, rockets, or any other device where the structural health is a concern.

This project explores the possibility of using the vibration response of a system to identify both linear and nonlinear damage in a multiple DOF system. While the physical design of this 5-DOF system is more analogous to a lab experiment than a real system, its dynamic behavior closely resembles that of a jointed structure. Each joint has a stiffness, and when damaged, the stiffness may change in a linear or nonlinear manner. This project presents a method of detecting damage in the joints of a multiple degree-of-freedom system by analyzing the vibration response of that system.

2 EXPERIMENTAL PROCEDURE

2.1 Project Setup

The 5-DOF system for this project consisted of five masses, connected by four springs. The masses and stiffnesses of the original system components are given in Table 2.1. The masses of the springs are included in the given mass terms.

Table 2.1: Original System

Item	Mass (kg)	Stiffness (N/m)
Mass 5	0.1642	
Spring 4		2626.903
Mass 4	0.06695	
Spring 3		11383.25
Mass 3	1.30345	
Spring 2		25568.52
Mass 2	0.28675	
Spring 1		56390.85
Mass 1	6.87075	

The system was suspended vertically, with a rod constraining lateral motion, as shown in Figure 2.1. Mass 1 is the base mass, and Mass 5 the top. The springs are numbered in the same manner, with the Spring 1 being the lowest spring, and Spring 4 the highest. A shaker was attached to the bottom of Mass 1 with a threaded rod. The spring-mass system was suspended by elastic cords connected to the bottom mass. The purpose of the cords was to support the weight of the system, preventing damage to the shaker, without inhibiting the motion of the system. A uniaxial accelerometer with a nominal sensitivity of 10 mV/g was attached to each mass, and a force transducer with a nominal sensitivity of 2.25 mV/g was located at the joint between the shaker and Mass 1.

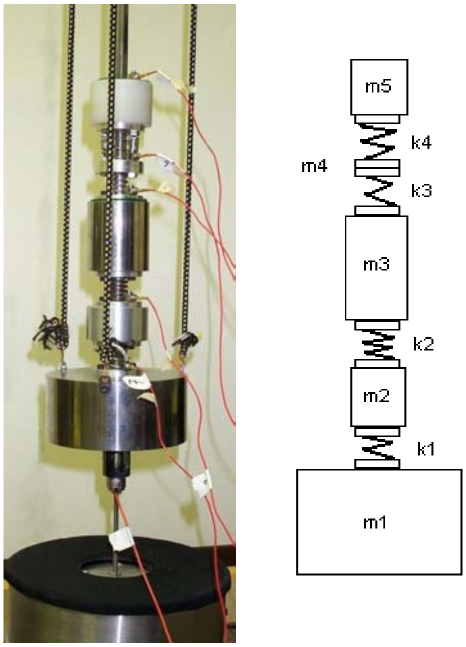

Figure 2.1: 5-DOF System setup.

2.2 Testing method

To find the vibration response for each mass, the system was excited with a shaped random input and the acceleration of each mass was recorded. To alleviate noise problems in areas of low amplitude in the response, each linear trial was completed in two runs. In the first run, the source had frequency content from 5-120 Hz. A second run of approximately 100-300 Hz. was then made, based on the results of the low-range trial. During the analysis of the data, the two runs were “spliced” together just above the highest frequency in the first run. For the nonlinear comparison, though, the data was not spliced so that the high frequency harmonics and other resulting nonlinear content would not be overwritten. For the nonlinear cases, the random input included frequency content from 5-100 Hz in each trial.

2.2.1 Linear Changes

To create linear changes in stiffness, the lower two springs were replaced with ones of lower stiffness. Figure 2.2 shows a normal spring in the system.

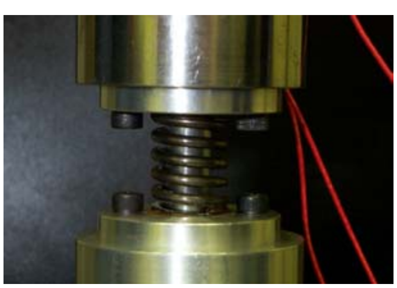

Figure 2.2: Spring and connection in system.

A testing schedule for these runs is shown in Table 2.2. The original system was run following the changes to quantify the variability caused by assembly and disassembly.

Table 2.2: Linear Testing Schedule

Run	Date	System	Splices
1	7/3/2001	original	
2	7/3/2001	original	105-300
3	7/3/2001	k1=35200.5 N/m	
4	7/3/2001	k1=35200.5 N/m	90-300
5	7/3/2001	k1=48335.01 N/m	
6	7/3/2001	k1=48335.01 N/m	105-300
7	7/3/2001	k2=21540.6 N/m	
8	7/3/2001	k2=21540.6 N/m	95-300
9	7/3/2001	k2= 9631.98 N/m	
10	7/3/2001	k2= 9631.98 N/m	95-300
11	7/5/2001	original	
12	7/5/2001	original	120-300

2.2.2 Nonlinear Changes

The first nonlinear change to the system came from introducing bumpers between Mass 4 and Mass 5. This was done by lowering the upper portion of the bumpers on their threaded rods and raising the lower section, as shown in Figure 2.3. This change resulted in a nonlinear increase in the stiffness of Spring 4.

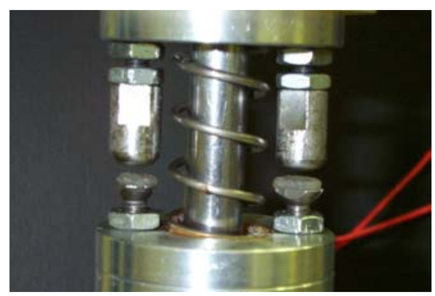

Figure 2.3: Nonlinear bumpers.

A second nonlinear change was introduced by removing the bolts securing Spring 2 and replacing them with pieces of threaded rod, as shown in Figure 2.4. This was first done with just the bottom set of bolts securing the spring to Mass 2, and then to the bolts securing it to Mass 3. This change prevented Spring 2 from going into tension, making its stiffness go to zero if Mass 3 moved further up than Mass 2.

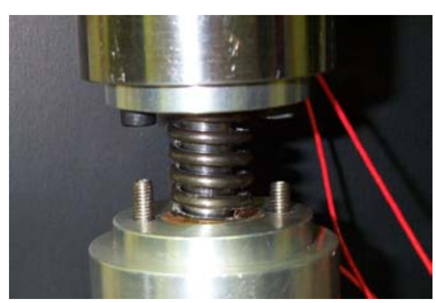

Figure 2.4: Nonlinear loose screws.

For both nonlinear changes, the system was run at various input amplitudes to analyze the effect of the magnitude on nonlinearities in the system. A testing schedule for the nonlinear runs is shown in Table 2.3.

Table 2.3: Nonlinear Tests

Run	Date	Amplitude	Comments
B1	7/10/2001	0.1	Bumpers
B2	7/10/2001	0.2	Bumpers
B3	7/10/2001	0.4	Bumpers
B4	7/10/2001	0.6	Bumpers
B5	7/10/2001	0.8	Bumpers
NB1	7/10/2001	0.1	No Bumpers
NB2	7/10/2001	0.2	No Bumpers
NB3	7/10/2001	0.4	No Bumpers
NB4	7/10/2001	0.6	No Bumpers
L2-1	7/12/2001	0.1	Loose at m2
L2-2	7/12/2001	0.2	Loose at m2
L2-3	7/12/2001	0.4	Loose at m2
L2-4	7/12/2001	0.6	Loose at m2
L3-1	7/12/2001	0.1	Loose at m3
L3-2	7/12/2001	0.2	Loose at m3
L3-3	7/12/2001	0.4	Loose at m3
L3-4	7/12/2001	0.6	Loose at m3

2.3 Data Acquisition

The data acquisition was performed using a DACTRON Spectrabook™, an eight channel 24-bit spectral analyzer, and the corresponding RT Pro™ software. The time responses, FRF's, and coherence data were all collected from the software. The FRF's were calculated using 20 averages at a sampling frequency of 640 Hz., while a Hanning window was applied to each average to reduce leakage.

3 DATA ANALYSIS

3.1 Frequency Response

FRF plots were computed for the theoretical model using the TFE function in Matlab™. From a time history acquired in Simulink™, TFE first windows the data, performs an FFT on the windowed sections, estimates the auto spectrum, and finally calculates the transfer function from the cross spectra and auto spectra. The experimental FRF's were similarly calculated by the DACTRON system. An important assumption made for the calculations is that the system is linear and time invariant [1].

3.2 Modal Analysis

For the 5-DOF system there are five natural frequencies, with five corresponding mode shapes. The first mode shape is the rigid body mode, corresponding to a 0 Hz natural frequency. The other four mode shapes correspond to the nonzero natural frequencies.

The experimental data was imported into Vibrant Technology's ME'scope™ in order to find the experimental mode shapes and natural frequencies. ME'scope™ uses curve fit estimation to find the mode shapes and natural frequencies from the FRF's.

The theoretical mode shapes were calculated by finding the eigenvalues and eigenvectors of the mass and stiffness matrices. The eigenvalues represent the natural frequencies while the eigenvectors, when mass normalized, represent the mode shapes [2].

Comparison between the theoretical and experimental mode shapes helped validate the theoretical model. Figure 3.1 shows the theoretical and experimental mode shapes for the original system.

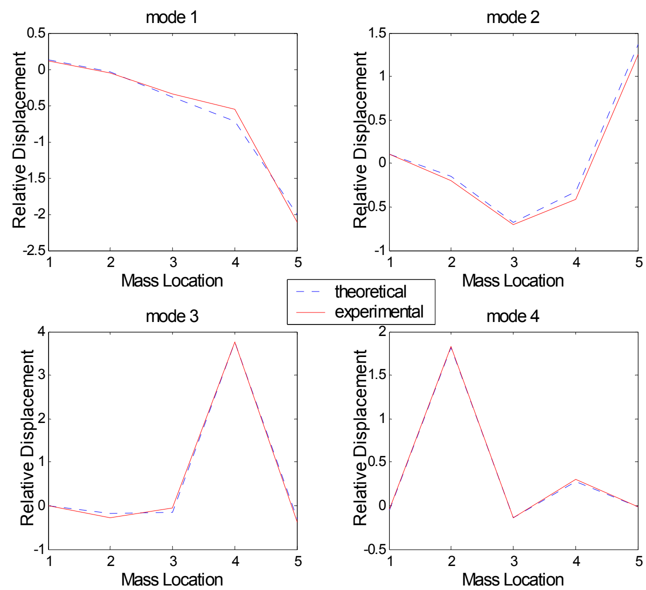

Figure 3.1: Mode shapes for original system

4 ANALYTICAL MODELS

In order to verify the experimental results, numerical models of the system were established in Simulink™.

4.1 Linear Model

For the linear model, the equations of motion were put into matrix form:

$$[M]\ddot{X} + [C]\dot{X} + [K]X = [F] \qquad 4.1$$

The block diagram for the linear model was derived directly from this equation.

4.2 Non-linear Model

For the nonlinear system, the matrix form of the equations of motion could not be used. A block diagram segment for each mass had to be derived and formed. Switch blocks in Simulink™ were used to represent the nonlinearities that were added to the physical system.

5 COMPARISON METHODS

5.1 Linear Changes

5.1.1 FRF's

The FRF's of the system were used to visualize the changes imparted to the system resulting from modification of the spring stiffnesses. They give an overall view of the system, but do not help pinpoint the location of the changes made to it [3]. Figure 5.1 shows the FRF's for the original system.

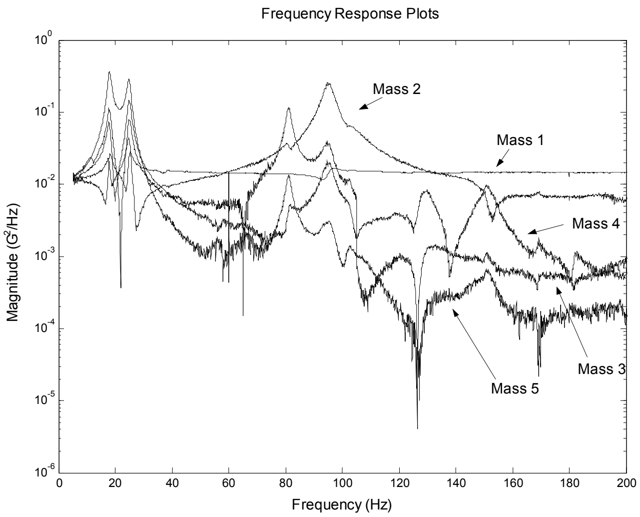

Figure 5.1: Experimental frequency response functions for the original system.

The FRF's were also useful for gauging whether the original system was truly linear. Often, for a nonlinear system, harmonics of the natural frequencies are visible on the FRF [3]. This feature of a nonlinear system was observed for the 5-DOF system, as harmonics were observed for the fourth natural frequency of the system. Since the mode shape corresponding to this frequency indicated that Mass 2 experienced the greatest motion, the non-linearity in the original system was most likely related to the motion of Mass 2.

5.1.2 Natural Frequencies

Changing different stiffnesses in the system brought about many changes in the response, but the most obvious change was in the natural frequencies. To evaluate this change, the difference between the new natural frequencies and those of the original system were calculated and plotted. Figure 5.2 shows these differences plotted against the mode number for the four non-zero natural frequencies in the system. The largest changes occur at the first and fourth frequencies for changes in the stiffness of Spring 1 or 2. When the stiffness of Spring 1 was reduced, the greatest change occurred in the fourth natural frequency. Lowering the stiffness of Spring 2 varied the first and fourth frequencies, but the change observed in the fourth frequency is almost twice the change in the first frequency. Included in the plot is the difference in frequencies for the two separate runs of the original system. The difference in frequencies between those runs is much less than the difference for the cases where the springs were changed; showing that changing the springs does make a significant difference.

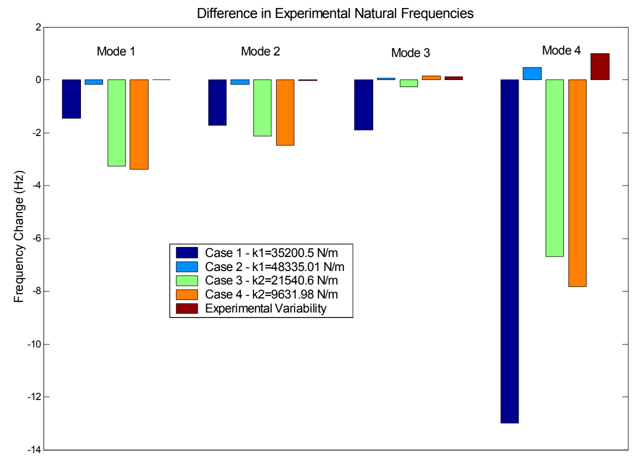

Figure 5.2: Frequency differences.

5.1.3 Mode Shapes

The mode shapes are valuable for illustrating the behavior of each mass at the natural frequencies. To evaluate the changes imparted to the system, the difference between the new mode shapes and those of the original system were calculated and plotted. When replacing the first spring with a less stiff spring, modes three and four were affected the most. The shape of modes one and two were most affected by a change to the stiffness of Spring 2. The experimental variability was also plotted, showing that the effects of changing the springs is significant.

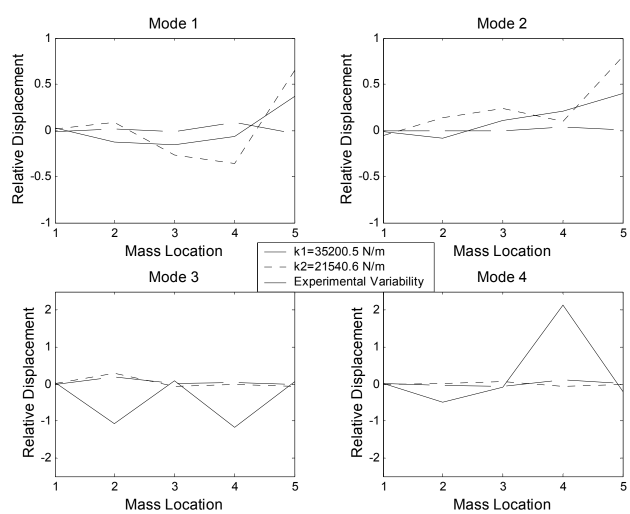

Figure 5.3: Experimental mode shape differences.

5.2 Nonlinear Changes

For the nonlinearities added to the original system, the FRF's, natural frequencies, and mode shapes did not change greatly. Different metrics were needed to identify the nonlinearities in the system. This identification was done using the power spectra and probability density functions.

5.2.1 Power Spectra

In the frequency domain, the power spectra of an ideal impact looks like a horizontal line [2]. The power spectra of an impacting nonlinear system should appear fairly horizontal, even past the highest excitation frequency. This phenomenon was used in this system as a way to identify nonlinearities. For both types of nonlinearities, the power spectra of the accelerations of the masses closest to the nonlinearities contained more high frequency content than the linear system. This response was observed for both the theoretical and experimental data.

More high frequency content is present in the power spectra for Mass 4 and Mass 5, which are located nearest to the non-linearity in the bumper case. This result is shown for both the theoretical and experimental systems in Figures 5.4 and 5.5, respectively.

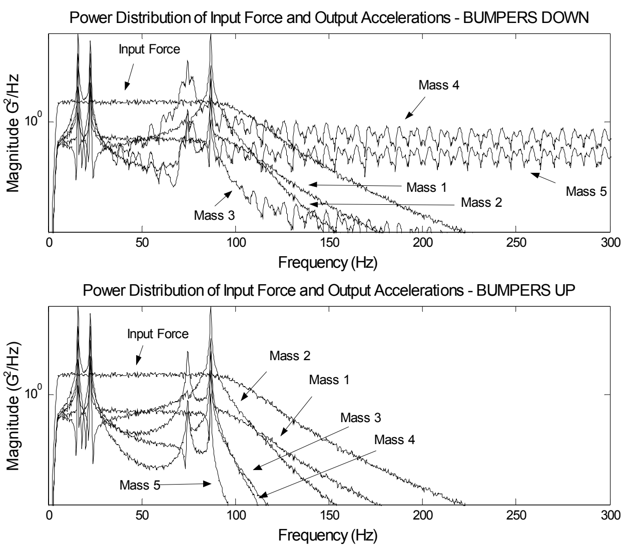

Figure 5.4: Theoretical power spectra for bumper system.

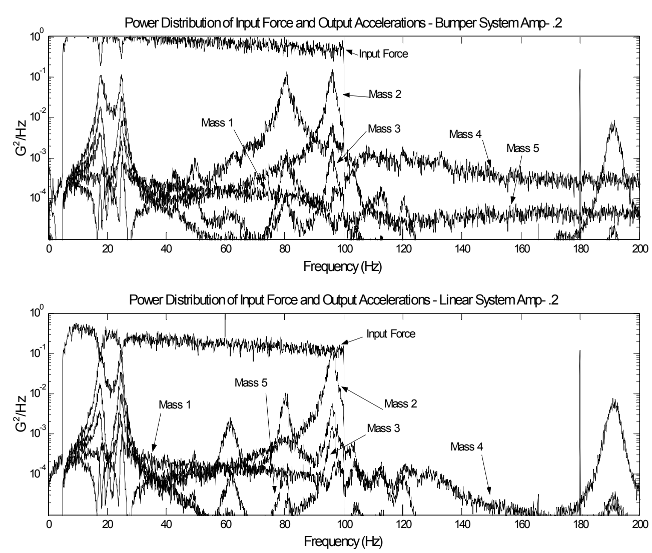

Figure 5.5: Experimental power spectra for bumper system.

The loose model gave the same results as the bumper model, with higher frequency content near the non-linearity, especially in the power spectra of Mass 2 and Mass 3. This result is shown for the theoretical and experimental cases in Figures 5.6 and 5.7, respectively.

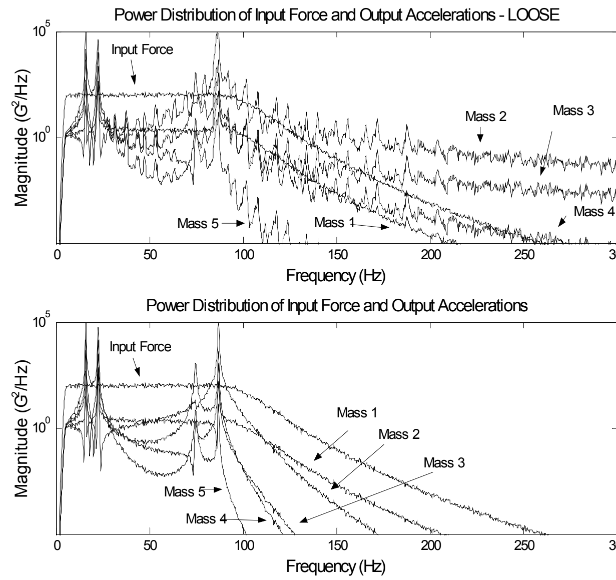

Figure 5.6: Theoretical power spectra for loose system.

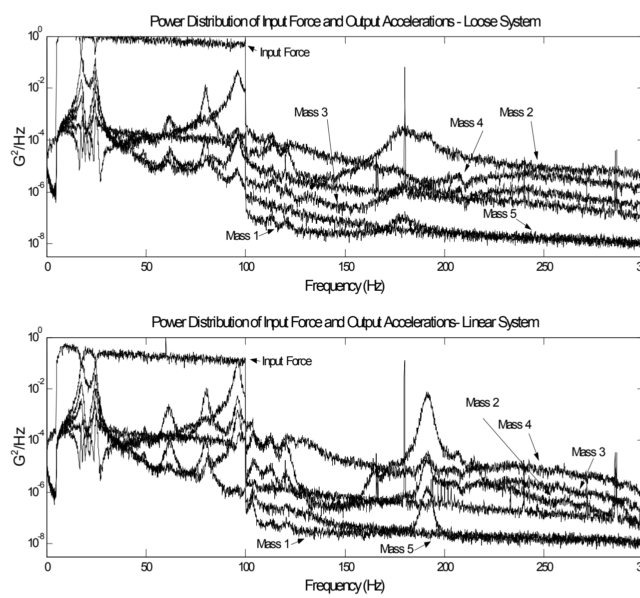

Figure 5.7: Experimental power spectra for loose system.

5.2.2 Probability Density

Deviation from a Gaussian probability density function (PDF) is often the result of nonlinear damage in a system [3].

Based on this fact, the difference between the PDF of the nonlinear runs and a Gaussian distribution was calculated and plotted to help identify nonlinearities. The results showed that for the linear system, the PDF was very close to Gaussian. Both nonlinear systems, however, showed deviation from a Gaussian distribution at the location of the nonlinearities. Figures 5.8 and 5.9 show a larger deviation from a Gaussian distribution in the PDF of Mass 4, which was located closest to the non-linearity of the bumper system.

Figures 5.10 and 5.11 show the same results for the loose system. The PDF of Mass 2, which is closest to the non-linearity, deviates more from a Gaussian distribution when the non-linearity is introduced.

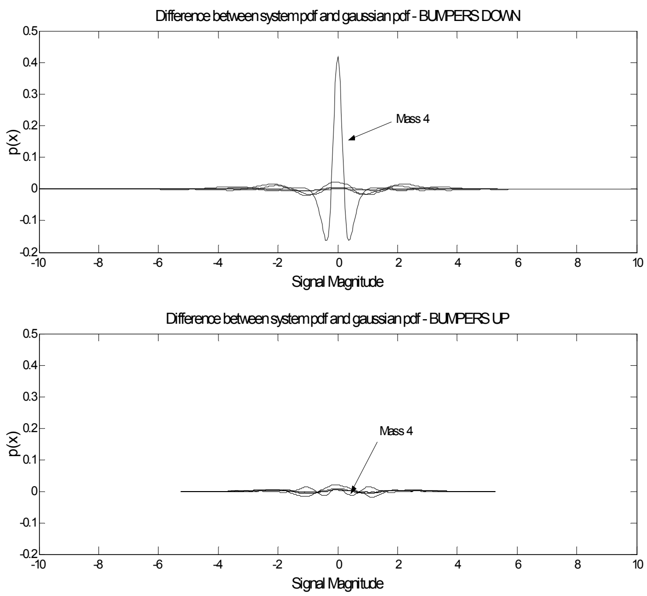

Figure 5.8: Theoretical PDF for bumper system.

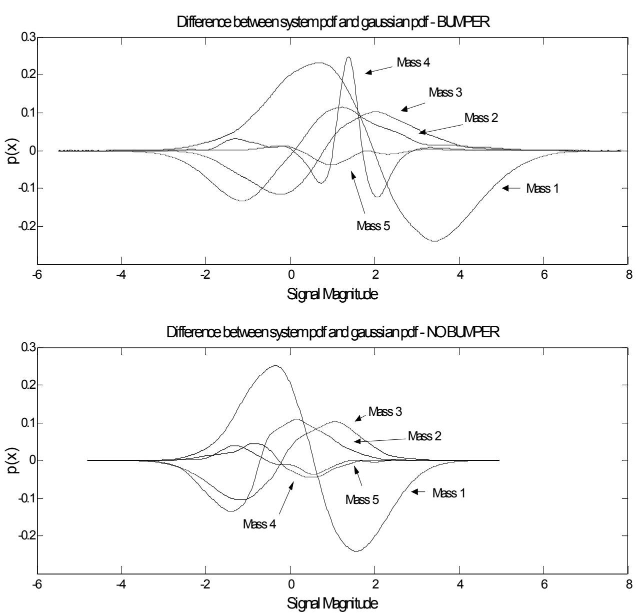

Figure 5.9: Experimental PDF for bumper system.

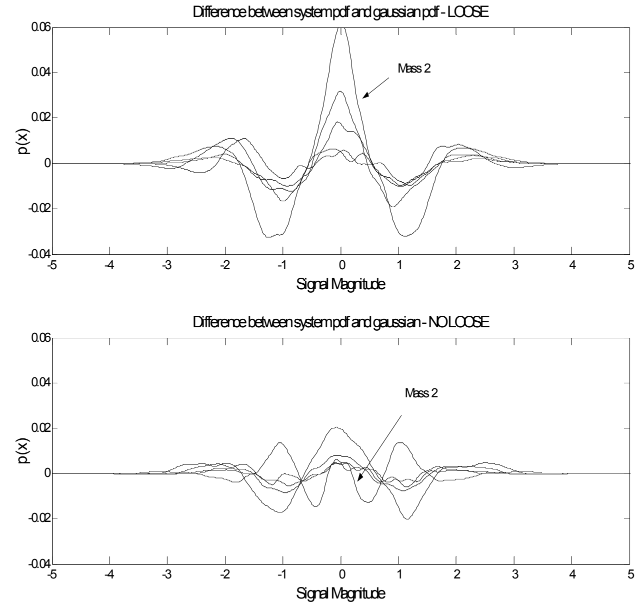

Figure 5.10: Theoretical PDF for loose system

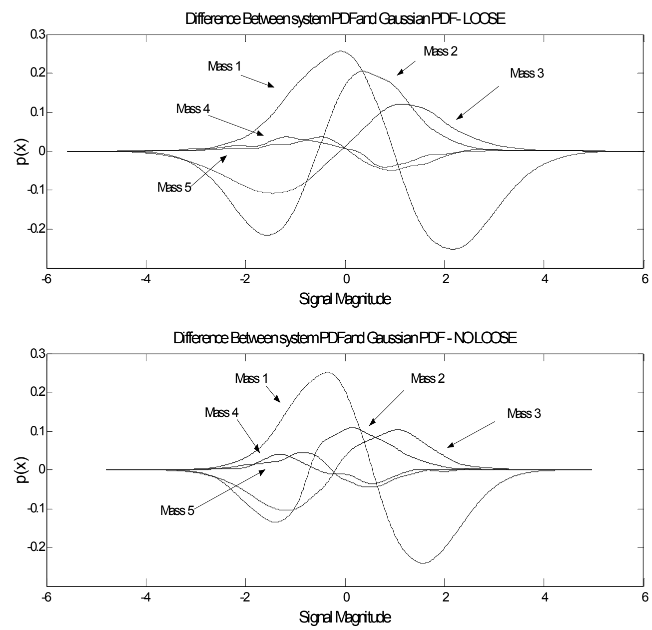

Figure 5.11: Experimental PDF for loose system.

5.2.3 FRF's

While linear stiffness changes resulted in shifts in the natural frequencies, nonlinear changes such as bumper collision and bolt loosening did not produce consistent changes. Both nonlinearities produced high levels of noise on the FRF plot lines. The FRF plots also showed locations of the nonlinearities because the FRF's for the masses were noisiest near the nonlinearities. The FRF's were useful for generalizing the presence of a non-linearity, but the power spectra and probability density functions were more powerful for pinpointing them.

6 CONCLUSIONS

For multi-DOF systems, linear changes to the stiffness mainly impact the natural frequencies and mode shapes. This result could easily be seen in the FRF's for the 5-DOF system in this project. In general, decreasing the stiffness of any spring resulted in lowering the natural frequencies of the system. For the 5-DOF system, changes to the stiffness of the first spring lowered the fourth natural frequency and changed the shape of the third and fourth modes. Changes in the stiffness of Spring 2 had a greater effect on the first two modes, and the first and fourth natural frequencies. Although these changes were seen, no concrete method of identifying the location of the linear change was determined.

For nonlinear changes to the system, the FRF's did not change noticeably. The changes were detected, however, by examining the power spectra and probability density functions of each object in the system. The masses closest to the non-linearity show more high frequency content in their power spectra than they would show in a linear system. The PDF's of the masses closest to the nonlinear diverge from a Gaussian distribution, and are another good way of identifying non-linearity in the system.

If the experiment were run again, it would help to try different types of inputs. A stepped sine input would help single out some of the nonlinearities inherent to the system. More time could also be spent on identifying and eliminating some of the nonlinearities in the original system. The stiffness of some of the springs may be bilinear, causing harmonics of the natural frequencies, but there was not enough time to investigate this phenomenon.

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[3] Farrar, C.R., Doebling, S.W., and Duffey, T.A., "Vibration-Based Damage Detection," presented at the SD2000 Structural Dynamics Forum, April 11-17, 1999.